Pol. J. Appl. Sci., **2016**, *2*, 16-20

THE CONSTRUCTION OF DIFFERENTIAL EQUATIONS OF MOTION PROGRAM FOR MULTI-PLANAR POSITIONING SYSTEM

Sviataslav E. Karpovich, Igar V. Dainiak, Dmitri G. Behun

Faculty of Computer Systems and Networks
Belarusian State University of Informatics and Radioelectronics, Minsk, Belarus

E-mail: dainiak@bsuir.by

Abstract: The subject of the article are the spatial electromechanical multi-coordinate systems. These systems are intended to determine complex motions on several axes simultaneously without mechanical elements of the transformation of a motion by means of ample opportunities of planar linear stepping motors (PLSM) with air bearing. Such PLSM allows the disscusion of the concept of the multi-coordinate electric drives which consists in constructive integration of mobile parts of several coordinates in one execution multi-coordinate system controlled by a digital computer. The method of construction of the motion equations based on the solution of the inverse dynamics problems was proposed.

Key words: program motion, precision positioning system, inverse dynamics problems, differential equations.

Introduction

Let us consider spatial electromechanical systems which cause complex motions along several axes simultaneously without mechanical elements of transformation of a motion. Kinematic units are completely excluded due to the use of electromechanical modules with air bearing. These modules have unlimited opportunities of electrical splitting of motion step and allow the scaling of motion characteristics. Mechanical connections are replaced by controlled electromagnetic constraints; it provides for achieving the highest parameters of accuracy, repeatability, velocity and has the constant metrological characteristics of a motion at long operating.

The electrical splitting of motion step by means of digital-to-analog control synthesizes a general motion from arbitrarily small segments, and the length of the segment is not dependent of the design of an electromechanical converter or kind of motion. In the case of a rotary motion this principle leads to the exclusion of mechanical reducing units by replacing its electrical conjunctions with an opportunity of motions at a wide range of speeds.

For complex motions with several degrees of freedom this principle of construction of coordinate systems allows the proposition of the concept of the construction of multi-coordinate drives [1–3]. The basic idea of the concept consists in the constructive integration of mobile parts of several coordinates in one execution multi-coordinate system. The integration of constructive elements assumes a division of channels of management by a complex motion and modular fulfillment of active elements of the electromechanical coordinate device. Using this approach it is possible to replace mechanical constraints by electromagnets, which are controlled by means of electronic management from a digital computer. The increase of functional opportunities of a drive can be achieved by means of number and the combination of typical electromechanical modules with the unification of control of a complex motion on all axes simultaneously.

The base module constructions of linear, rotary and planar types are primary elements for the building of a coordinate system. Combining them into complex modules we can get a spatial motion system which recognizes complex movements with given properties. In the paper we'll consider some types of positioning systems and construction of program motions.

Planar linear stepping motor and multi-planar positioning system

A planar linear stepping motor (PLSM) with separated coordinates and air-magnetic bearings is shown in Fig. 1; it provides motions along ortogonal X and Y axes.

The motionless stator 1 has three ortogonal to each others' teeth zones. Inductor 2 consists of three groups of electromagnets (marked with dashed lines) which are separated in space and united by one frame. The inductor's teeth structures are formed on block poles according to the three teeth zones of the stator. The gap between inductor and stator is made by pressed air transferred through capillary holes.

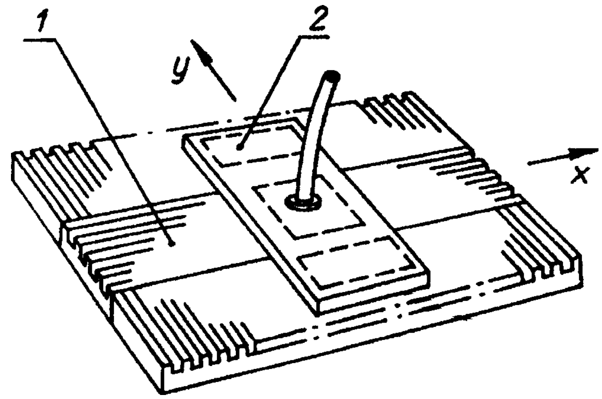

Fig. 1: Planar linear stepping motor with separated coordinates: 1 – stator; 2 – inductor.

A planar linear stepping motor (PLSM) with combined coordinates is shown in Fig. 2.

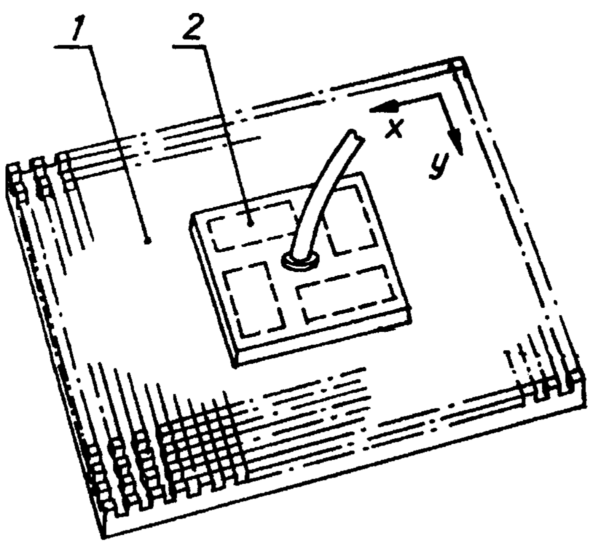

Fig. 2: Planar linear stepping motor with combined coordinates: 1 – stator; 2 – inductor.

Using the elementary modules recognizing one-coordinate and two-coordinate motions, flexible industrial systems can be created; their motion systems are the base of robotics for technological equipment and result in the realization of any motions in 3D space.

The robotic complex for the forming of wire interconnections is shown in Fig. 3; it is intended for the automation of one of the basic and the most complex operation within the technological process of assembly of the integrated circuits in microelectronics.

The multi-robot system (Fig. 3) is built on the basis of one-coordinate and two-coordinate PLSM, which have high accuracy and high speed. The system can perform most assembly operations in a common workspace: loading components, the forming of wire interconnections, the controlling of parameters with a contact method, removals of ready products, etc. This complex permits not only program installation of all wire interconnections of a particular circuit but also programmed transition to other types of products.

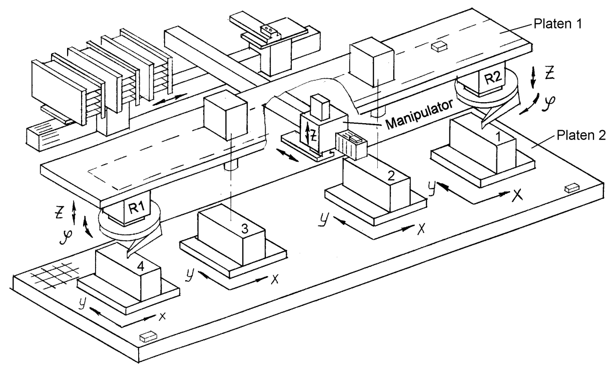

Fig. 3: Multi-robot system for the forming of wire interconnections in the assembly of integrated circuits: 1,2,3,4 – x, y planar robots.

Mathematical description of planar linear stepping motor

The basic problem of mathematical modelling of multi-coordinate systems is the problem of the construction of the required program motions by the definition of control actions. This problem in mathematical formulation is reduced to the choosing of parameters of differential equations of a motion or to defining unknown parts of the differential equations using the condition of the existence of the given particular solutions. In more general formulation the problem of construction of required program motions is reduced to the construction of the differential equation system using a priori knowledge of the properties of required motions, which are described by this system. Generally speaking, the solution to this problem is not unequivocal, and that results in the constructing of the required motions using additional conditions for electromechanical coordinate system.

To get the basic laws for electromechanical coordinate systems on a basis PLSM, we can use various mathematical models, which describe stability, quality of motion, velocity and range of working speeds, and the dynamics of a system [4, 5].

The mathematical model of the two-coordinate stepping electric drive presented in Fig. 1 and Fig. 2 with bi-phase excitation and three degrees of freedom (ortogonal linear x, y and angular φ – rotation of inductor in a plane of a stator) can be written as following system [6, 7]:

$$\begin{cases} m\dfrac{d^2x}{dt^2} + \beta_x\dfrac{dx}{dt} + F_{cx} = F(i_{1x}, i_{2x}, \varphi, t) + F_x(t), \\ m\dfrac{d^2y}{dt^2} + \beta_y\dfrac{dx}{dt} + F_{cy} = F(i_{1y}, i_{2y}, \varphi, t) + F_y(t), \\ I\dfrac{d^2\varphi}{dt^2} + \beta_\varphi\dfrac{d\varphi}{dt} + F_{c\varphi} = M(t), \\ r_{1x}i_{1x} + \dfrac{d\psi_{1x}}{dt} = U_{1x}(x, \varphi, t), \\ r_{2x}i_{2x} + \dfrac{d\psi_{2x}}{dt} = U_{2x}(x, \varphi, t), \\ r_{1y}i_{1y} + \dfrac{d\psi_{1y}}{dt} = U_{1y}(y, \varphi, t), \\ r_{2y}i_{2y} + \dfrac{d\psi_{2y}}{dt} = U_{2y}(y, \varphi, t), \end{cases} \tag{1}$$

where:

m – distributed mass (the load and the inductor);
x, y – linear coordinates in the plane of a motion;
$\beta_x, \beta_y, \beta_\varphi$ – factors describing viscous friction;
$F_{cx}, F_{cy}, F_\varphi$ – resistance forces on x, y, φ accordingly;
φ – coordinate describing angular displacement of inductor in the plane of a motion;
$F(i_{1x}, i_{2x}, \varphi, t), F(i_{1y}, i_{2y}, \varphi, t), M(t)$ – the force characteristics of a drive accordingly on coordinates x, y, φ;
r, i, ψ – resistance, current and inter-linkage of phase windings correspondingly;
$F_x(t), F_y(t)$ – external mechanical influence on coordinates x and y;
I – moment of inertia of distributed mass relatively to ortogonal axis to the plane of motion;
$U_{1x}, U_{2x}, U_{1y}, U_{2y}$ – laws of voltage in phase windings (functions of control action).

The systems with similar structure of the equations can be written for any multi-coordinate drive based on a PLSM [8]. The combined equations (1), which describe the physical processes within stepping electric drive, are a complete mathematical model of the considered device. The various representations of this model are used for mathematical research depending on the particular purpose. In our case for researching the dynamics of a coordinate system and for the constructing of program motions it is convenient to use the complete mathematical model (1) in the form

$$\ddot{x}_i = f_i(x, \dot{x}, t), \qquad i = 1, \ldots, n; \tag{2}$$

where:

$x(x_1, ..., x_n)$ – vector of generalized coordinates of system;
$\dot{x}(\dot{x}_1, ..., \dot{x}_n)$ – vector of generalized velocities of system.

Construction of the motion equations based on the solution of the inverse dynamics problems

The inverse problems of dynamics are the definition of active forces applied to the mechanical system, parameters of system and in addition imposed on them. They constrain, refer to as, at which the motion with given properties is one of possible motions of considered mechanical system. Thus, the properties of a motion can be given in different ways, for example as quantitative and qualitative restrictions for coordinates and speeds of a motion as invariant parities.

The works of the different authors are devoted to the solution of inverse dynamics problems [9–11]. Here we stop at the basic theoretical preconditions, which are necessary for the solution of problems of constructing program motions.

We assume that properties of a motion of a mechanical system, which are defined by a vector $x(x_1, ..., x_n)$ of generalized coordinates and vector $\dot{x}(\dot{x}_1, ..., \dot{x}_n)$ of generalized velocities, are given as variety

$$\omega_\mu(x, \dot{x}, t) = c_\mu, \qquad \mu = 1, ..., m \leqslant n. \tag{3}$$

Concerning functions ω_μ we assume, that the equality $\omega_\mu(x, \dot{x}, t) = c_\mu$ are joint and are independent in some part of a phase space $G\{x, \dot{x}\}$ at $t \geqslant t_0$.

The given variety of properties of a motion is in essence the integrated variety of the appropriate equations of a motion of considered mechanical system. Naturally, therefore, for the solution of inverse problems of dynamics it is necessary to construct the equations of a motion of considered mechanical system on given integrated variety, so that the expressions $\omega_\mu(x, \dot{x}, t) = c_\mu$ are the integrals of these equations. Furthermore, from the constructed equations it is necessary to determine the required generalized forces, parameters and connections exhibiting a motion with given properties (3).

In special cases, when the structure of equations of a motion is known but required additional forces and parameters of considered mechanical system for recognizing a motion with given properties are unknown, it is necessary to determine the equations of a motion on a given integrated variety and to find required unknown equations [12].

When a part of the equations of a motion of considered mechanical system is a priori, it is necessary for the solution of the inverse problems of dynamics to build the missing equations on the given integrated variety and to determine the required generalized forces, parameters and interrelations, which allow the realization of a motion with the given properties. Thus, the solution of the inverse problem of dynamics in sufficiently general mathematical interpretation is reduced to the construction of the motion equations of mechanical system on a given integrated variety as properties of required motion. In addition, the motion equations are necessary to be defined in the form

$$\ddot{x} = X(x, \dot{x}, t). \tag{4}$$

The construction of differential equations can be carried out using the Erugin's method [13]. According to this method, at first the necessary and sufficient conditions must be satisfied that the given integrals form the integrated variety of the differential equation system. These conditions can be obtained by comparing the time derivatives of given integrals in the form of the required equations of arbitrary functions, which are equal to zero on a given integrated variety.

In our formulation the conditions for feasible motion with given properties (3) can be written as

$$(\mathrm{grad}_{\dot{x}}\omega_\mu \cdot X) = R_\mu(\omega, x, \dot{x}, t) - \varphi_\mu; \quad \mu = 1, ..., m, \tag{5}$$

where

$\varphi_\mu = (\mathrm{grad}_x\omega_\mu \cdot \dot{x}) + \frac{\partial \omega_\mu}{\partial t}, R_\mu(\omega, x, \dot{x}, t)$ – functions, which are identically equal to zero at $c_\mu \neq 0$ and any at $c_\mu = 0$, and equal to zero on integrated variety Ω, for example, they can be holomorfic functions of variables $\omega_1, ..., \omega_n$ in area Ω_ε at $t \geqslant t_0$, which have members not less than first order in the decomposition on degrees of these variable.

The received equations (5) are the equations for the defining the right hand side of equations (2).

In the case of $m = n$, we can find the required equations directly solving the equations (5):

$$\ddot{x}_\nu = \sum_{i=1}^{n} \frac{\Delta^{i\nu}}{\Delta}(R_i - \varphi_i), \tag{6}$$

where

$\Delta = \left|\frac{\partial \omega}{\partial x}\right|_m^m \neq 0$;
$\Delta^{i\nu}$ – algebraic adjunct for element i of the determinant Δ.

If $m < n$, it is more convenient to find the vector-function X of the right parts of the equations as a sum,

$$X = X^\nu + X^\tau, \tag{7}$$

where vector X^ν is ortogonal to variety $\Omega_{\dot{x}}\{\omega(x, \dot{x}, t)_{x=in\nu} = 0\}$, and it can be found up to the Lagrange multipliers:

$$X^\nu = \sum_{i=1}^{m} \lambda_i \mathrm{grad}_{\dot{x}}\omega_i,$$

and the vector X^τ is a component of vector-function along variety $\Omega_{\dot{x}}$, it is determined by a condition

$$(\mathrm{grad}_{\dot{x}}\omega_\mu \cdot X^\tau) = 0; \mu = 1, ..., m. \tag{8}$$

By substituting vector-function X in the form (7) into conditions of feasible motion (5), we get

$$(\mathrm{grad}_{\dot{x}}\omega_\mu \cdot X^\nu) = R_\mu - \varphi_\mu, \tag{9}$$

and taking X^τ into account we get

$$\lambda_i = \frac{1}{\Gamma} \sum_{j=0}^{m} \Gamma_{ij}(R_j - \varphi_j); \quad i = 1, ..., m,$$

where

$\Gamma = |\mathrm{grad}_{\dot{x}}\omega_i \cdot \mathrm{grad}_{\dot{x}}\omega_j|_m^m \neq 0, \Gamma_{ij}$ – algebraic adjunct of element i, j of the determinant Γ.

Thus, we finally get

$$X^\nu = \frac{1}{\Gamma} \sum_{i,j}^{1,m} \Gamma_{i,j}(R_j - \varphi_j)\mathrm{grad}_{\dot{x}}\omega_j.$$

The components of vector-functions X^τ are determined by solving a system of the linear equations (8) and can be written as

$$X_r^\tau = -\sum_{s=m+1}^{n} D^{rs}Q_s, \qquad r = 1, ..., m,$$

where

$X_s^\tau = DQ_s$;
D^{rs} – the determinant, which is received by replacing its column i to column s of a matrix $\left(\frac{\partial \omega}{\partial \dot{x}}\right)_n^m$;
$Q_s = Q_s(x, \dot{x}, t)$ – any functions.

So, the required equation system (1) can be written in the following form:

$$\begin{cases} \ddot{x}_r = \dfrac{1}{\Gamma} \displaystyle\sum_{i,j}^{1,m} \Gamma_{i,j}(R_j - \varphi_j)\frac{\partial \omega_i}{\partial \dot{x}_r} - \sum_{s=m+1}^{n} D^{rs}Q_s, \\ \quad r = 1, ..., m, \\ \ddot{x}_s = \dfrac{1}{\Gamma} \displaystyle\sum_{i,j}^{1,m} \Gamma_{i,j}(R_j - \varphi_j)\frac{\partial \omega_i}{\partial \dot{x}_s} + DQ_s, \\ \quad s = (m+1), ..., n. \end{cases} \tag{10}$$

As we see, the solution of a general problem of construction of the motion equations contains unknown functions $R_j(\omega, x, \dot{x}, t)$ and $Q_s(x, \dot{x}, t)$, which are not considered in this paper. Surely, these functions must be chosen so that conditions of existence and uniqueness of the solution of combined equations (10) in area Ω_ε are satisfied.

The constructed system of the equations (10), which allows the realization of a motion with given properties (5), can be presented in the form of a vector equation

$$\ddot{x} = \frac{1}{\Gamma}\sum_{i,j}^{1,m}\Gamma_{i,j}(R_j - \varphi_j)\mathrm{grad}_{\dot{x}}\omega_i + X^{\tau}, \qquad (11)$$

where vector X^{τ} is determined by conditions (8).

We obtained the solution in a broad statement of the problem using the universality of method, and the received solution can be used for construction of the equations of a motion in many inverse problems of dynamics.

We notice, when we solve of inverse problems of dynamics in some special cases, it is expedient to construct equations of a motion using only some of given integrals at first, and then to build missing equations using the remaining given integrals.

Conclusions

1. The modular approach for designing and creating positioning systems, the development of the concept of constructive integration of mobile parts of several coordinates in one execution multi-coordinate electromechanical converter, which include the mathematical description of processes in the multi-coordinate drive, method for synthesis of parameters and mechatronic systems are proposed.
2. In received expressions for control action there are some functions, equal to zero at the motion program if the motion occurs without deviations from the given. If there is the deviation from the program, control action is necessary to be found in view of stability.
3. A development of the common theory of construction of multilevel multi-coordinate robot systems devices by dynamic criteria based on solution of inverse dynamics problems is described.

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Received: 2016
Accepted: 2016