

IMAGE DITHERING USING CHAOTIC SIGNAL GENERATED BY LOGISTIC MAP

WOJCIECH KORNETA

*Computer Science and Automation Institute
Lomza State University of Applied Sciences, Lomza, Poland*

E-mail: wkorneta@pwsip.edu.pl

Abstract: The dithering of images with chaotic signal generated by logistic map is presented and compared with the dithering by rectangular and triangular noises. The brightness reduction of the original image with 256 grey levels to 4 grey levels is considered. The visual effect of dithering and quantization is presented, quantified and compared for different noise ranges. The best range for each noise is specified.

Key words: dithering, chaos, logistic map.

In recent years image processing techniques utilizing noise have attracted considerable attention [1–6]. One of these techniques frequently used when the colour resolution of an image decreases is dithering. In this technique a noise is added to an image before decreasing its resolution. This creates an illusion of high colour resolution in images whose colour resolution was reduced. Usually the noise with rectangular, triangular or normal probability distribution is used in dithering. In this paper we show the effect of adding a chaotic signal generated by logistic map [7, 8] to an image before decreasing its resolution. The logistic map is nonlinear function generating successive values by iteration. These values form many different chaotic signals depending on the value of the parameter of logistic map.

In Fig. 1 we show the image of the function sine quantized to 256 grey levels. This image is formed by pixels arranged in 256 columns and rows. Their brightness changes from 0 to 255.

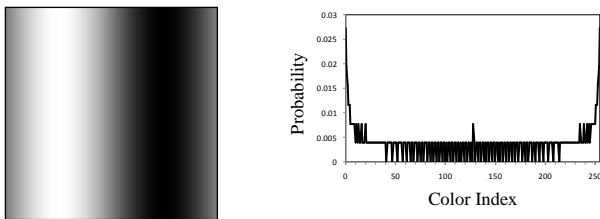


Fig. 1: The original image of the function sine with 256 grey levels and its histogram.

By rounding grey levels of the image in figure 1 to 4 grey levels, one gets the image shown in Fig. 2. The distance between these 4 grey levels is $\Delta = 85$. There are flat areas of brightness in this image which do not represent continuous changes of the function sine correctly.

In order to remove flat areas of the brightness and create an illusion of continuous brightness, various random

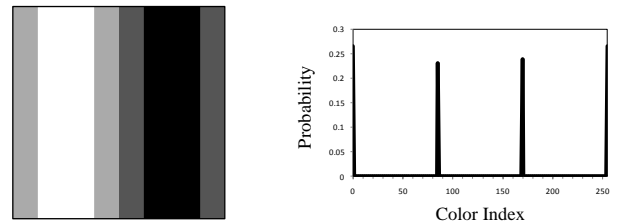


Fig. 2: The image shown in Fig. 1 after rounding its grey levels to 4 levels and its histogram.

numbers are added to the pixels grey levels before rounding them to a lower number of grey levels. In this paper we consider a chaotic sequence of numbers generated by logistic map defined by the following equation [7, 8]:

$$x_{i+1} = Rx_i(1 - x_i) \quad (1)$$

where $i=0, 1, 2, 3, \dots$ and R is a parameter. For $R=4$ this map generates numbers in the interval $[0,1]$. The probability density function and cumulative distribution of these numbers after their rescaling $r \times (x_i - 0.5)$ to the range $r = 2 \times \Delta = 2 \times 85$ are presented in Fig. 3. In Fig. 3 we also present the triangular and the rectangular probability density function and their cumulative distributions. One can notice that these three distributions are very different.

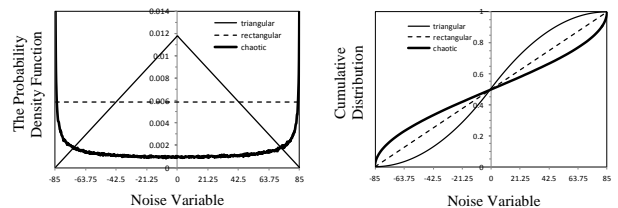


Fig. 3: The probability density function and cumulative distribution of the triangular noise, the rectangular noise and chaotic sequence with the mean zero and the range $[-85,85]$.

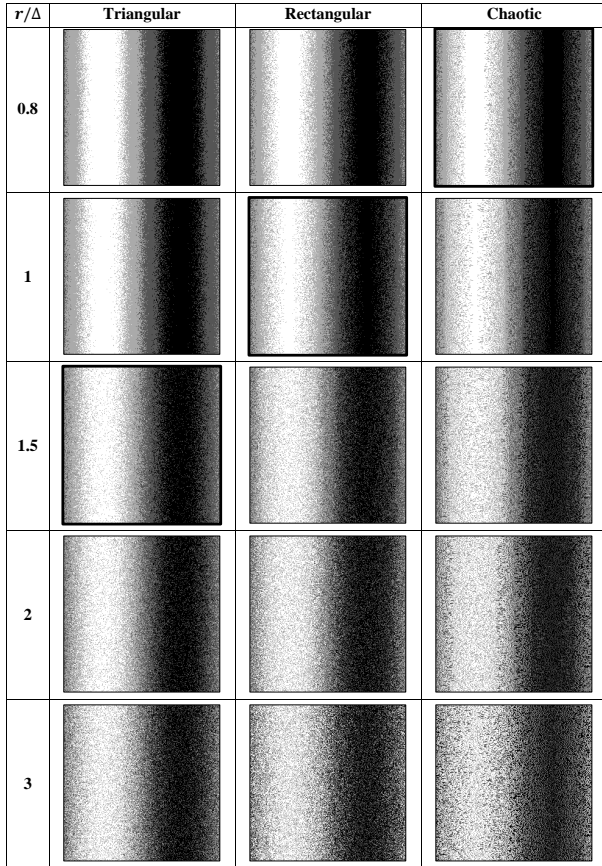


Fig. 4: The image shown in Fig. 1 with added triangular, rectangular and chaotic noises with a mean zero and the range r normalized by $\Delta = 85$ after its quantization to 4 grey levels. The best image for each noise is denoted by thick frame.

Random numbers in the range r for the rectangular and the triangular probability distribution are generated by the formulae $x_i = r \times (Rnd - 0.5)$ and $x_i = 0.5 \times r \times (Rnd_1 + Rnd_2 - 1)$ respectively, where Rnd denotes the random number generated in the range $[0,1]$ with uniform probability. The images obtained by adding triangular, rectangular and chaotic noises with a mean zero to the image shown in Fig. 1 before its quantization to 4 grey levels are shown in Fig. 4. We performed the dithering for different noise ranges. The visual effect depends on the relation between the noise range r and the distance $\Delta = 85$ between the four grey levels. In the original image the brightness changes only along the horizontal axis. The illusion of continuous brightness changes can thus be expressed by the row of pixels, where each pixel's brightness is the average of pixels grey levels situated in the same image column. Let's denote these average grey levels obtained for the original image and the image after dithering and quantization by $I_o(i)$ and $I_d(i)$ respectively, where $i=0, 1, \dots, 255$. The error quantifying the quality of visual effect we define as:

$$Error = \sqrt{\frac{\sum_i (I_d(i) - I_o(i))^2}{256}}. \quad (2)$$

The dependence of this error for the triangular, the rectangular and chaotic noises on the noise range is shown in Fig. 5. For low noise ranges the smallest error is for the chaotic noise generated by a logistic map, and this noise can replace frequently used rectangular noise. For high noise ranges the smallest error is for the triangular noise. The best images obtained by dithering for each noise are denoted by thick frame in Fig. 4. They approximate continuous brightness changes of the function sine much better than simple rounding of brightness shown in Fig. 2.

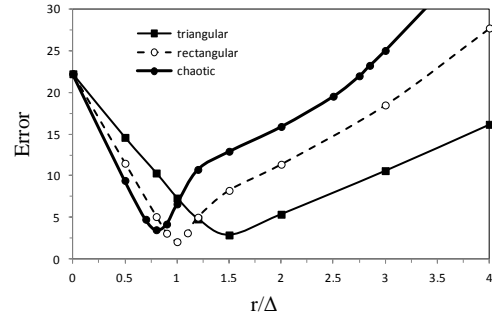


Fig. 5: The dependence of the error defined by the equation (2) between the original image and the image obtained by its dithering with different noises and quantization to 4 grey levels on the normalized noise range.

Literature

- [1] Ryu C., Kong S.G., Kim H. Enhancement of feature extraction for low quality fingerprint images using stochastic resonance. *Pattern Recognition Letters*, 32:107–113, 2011.
- [2] Jha R.K., Biswas P.K., Chatterji B.N. Image denoising using stochastic resonance. In *Proceedings of TENCON 2005–IEEE*, pages 343–348, 2005.
- [3] Jha R.K., Biswas P.K., Chatterji B.N. Contrast enhancement of dark images using stochastic resonance. *IET Image Processing*, 6:230–237, 2012.
- [4] Jha R.K., Chouhan R., Biswas P.K. Noise-induced contrast enhancement of dark images using non-dynamic stochastic resonance. In *Proceeding of National Conference on Communications (ncc)*, number 3, pages 1–5, 2012.
- [5] Ulichney R.A. Dithering with blue noise. *Proceedings of the IEEE*, 76:56–79, 1988.
- [6] Mohammad F. Image compression via dithering for enhanced images. In *International Conference on Electrical Engineering, ICEE 07*, pages 1–7, 2007.
- [7] May R.M. Simple mathematical models with very complicated dynamics. *Nature*, 261:459–67, 1976.
- [8] Sprott J.C. *Chaos and Time-Series Analysis*. Oxford University Press, 2003.

Received: 2016
Accepted: 2016